Approaches for mathematical modelling of grain separation

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Abstract
This paper presents a review of former approaches for mathematical modelling of grain separation in combine harvesters and proposes a new approach on the base of extended exponential functions. The model equations for the remaining unseparated grain versus the separation length as well as for the cumulative sum of already separated grain based on empirical as well as on statistical approaches are put together in two tables; one containing single exponential functions, the other approaches with two exponential functions, to better describe the delayed separation start at the front of the separation unit. In addition to that the extended convection diffusion model and an analytical approach are described.

Experimental data can be reproduced quite well by these equations, fitting the data by proper determination of the different threshing and separation coefficients. But the influences of crop properties, design and adjustment of the separation units on these coefficients have to be worked out in the future.

Keywords. Modelling, threshing, cleaning unit, straw walker, combine harvester

Introduction
Mathematical models to describe the separation process in grain harvesters may have the following advantages: reduction in test expenditure, improved understanding of fundamental relationships, hints for possible improvements, simulations of the influences of different parameters, and the estimation of possible performance increases. Different approaches exist in literature, which describe the separation process. Interestingly, the methods for describing the threshing process, the straw walker, as well as the cleaning system are similar. Aside from stochastic approaches, convection-diffusion and analytical models have been developed.

Methods
Test equipment
Mostly, laboratory experiments are the basis for developing models and for their later validation. During timed test runs, the grain is collected in boxes arranged under the separation unit, with one additional box also placed at the end.

Threshing and separation unit
At Hohenheim University a test stand is used for extended investigations on tangential threshing systems, Figure 1, (Büermann, 1996). It allows for free positioning of the feeder, beater, and the walker in relation to one or more threshing drums. Each element of the test stand can be powered independently of the other elements. Figure 1 shows a design with a second threshing drum. A conventional header with cross auger produces the same condition in feeding as on combines. The threshing system is 1.06 m wide, the drum diameter is 600 mm. The threshing system is followed by four straw walkers. Boxes are mounted below the threshing and separating system. The separated masses of the threshing unit are caught in the boxes 1 to 5, the separated masses from beater and walker are caught in the boxes 6 to 12.
material in the boxes is weighted and cleaned to quantify the part of grain and the part of MOG in each box. The walker overflow is caught in the box at the end. Threshing and separating losses are specified with laboratory threshing machines and cleaning units. Grain samples from the threshing and separating section are drawn for measuring the part of broken grain in the samples by computer vision and image analysing system (Schneider and Kutzbach, 1997). The speed and torque of the feeder, threshing drums, beater, and walker are measured for calculating the power consumption of the threshing and separation system. Sheaves harvested with a binder are put in a layer of 25 m length on a feeding belt of 2 m width in front of the test stand. The belt speed is 1.56 m/s. Different throughputs are realized by different amounts of sheaves per meter on the belt. The sheaves are fed with the ears first into the header to get a correct feeding of the threshing device. The sheaves are sliced and the material is shifted in a even layer.

\[ w_A \]

Figure 1. Test stand for threshing and separation research

Cleaning unit

To optimize design parameters and adjustments of the combine harvester cleaning units, as well as to improve grain separation and to increase the grain feedrate another test facility is used at Hohenheim University (Zhao, 2002).

The main components of the test stand are two oscillating frames, one for the shoe assembly (8) and another for the grain pans (9,10). They are driven by an electric motor with an adjustable transmission (1) via two crank drives (2, 4). The two drives are coupled together to allow for oscillation with the same frequency and the same phase or with a certain phase shift. The mechanical parameters, for instance the oscillation frequency and the angle of vibration, can be varied. The amplitude of the sieve and the grain pans can be adjusted separately by changing the crank radius of the drive from 15 to 40 mm. A sieve with a dimension of 1560 x 240 mm can be fixed in the oscillating frame for the shoe assembly. Four long links are used to suspend the sieve to achieve a quasi-sinus oscillation movement. The sieve is divided over its length into 5 sections. Each section is supplied with an air stream by one of 5 separately adjustable fans (6, A-E) via an exchangeable direction channel so that the air distribution along the sieve (air velocities \( w_A \) in the section A-E) and the air stream direction (20°, 30° or 40°) can be exactly adjusted.
The winnowing steps are realized between the grain pans and the sieve. The height of the winnowing steps can be varied from 50 to 200 mm by changing the mounting position of the grain pans in the frame. A long grain pan with a length of 900 mm is used in case of investigations of one single winnowing step.

The winnowing step between the sieve and the grain pan is defined as the first winnowing step; between the two grain pans as the second. Each of the adjustable fans (13, F-G) supplies a uniform air stream for each of the winnowing steps through an expansion chamber and an air channel. Two perforated steel plates ensure the uniformity of the air stream at the outlet. The direction of the air stream from the air channel can be changed from 10° to 30°.

![Diagram of the cleaning unit of combine harvesters with two winnowing steps](image)

**Figure 2. Test stand for the cleaning unit of combine harvesters with two winnowing steps**

Before a test run is started, the MOG is placed by hand onto the 14 m long conveyor belt (11) and the grain kernels are then applied automatically by a bucket-wheel grain feeder (12) on top of the MOG layer. The grain-chaff-mixture is further fed to the first grain pan through a sliding plate to avoid undesirable pre-segregation during the delivery. The grain feedrate can be easily varied from 0.5 to 5.5 kg/(s⋅m), which is expressed to 1 m sieve width. The grain and a part of the MOG are separated by the sieve and collected in the 10 collection boxes (5) under the sieve.

**Theory**

The grain mass $m_{Gi}$ of each collection box is measured and related to the total grain $m_{G\text{tot}}$. (Figure 3).
The separation rate $\delta_{Gi}$ is given by relating the separated grain fractions additionally to the length of the individual boxes:

$$\delta_{Gi} = \frac{m_{Gi}}{m_{Gtot} \Delta l_i} = \frac{m_{Gi}}{\Delta t m_{Gtot} \Delta l_i}$$

The grain distribution, which is the separation rate over the separation length, Figure 3, greatly characterizes the separation process. The cumulative sum of the separated grain $S_G$ is given by addition of the individual related grain quantities. The difference to 1 (100%) is the remaining unseparated grain $R_G$.

$$R_{Gi} = 1 - S_{Gi} \quad S_{Gi} = \sum_{i=1}^{j} \delta_{Gi} \Delta l_i$$

The separation efficiency $\eta_{G}$ denotes the grade of separation. The separated grain quantity for each section is related to the amount of grain delivered to this section.

$$\eta_{G} = \frac{\delta_{Gi}}{R_{G(i-1)}}$$

The local separation rate $\delta_{G}$ is the negative derivative of the remaining unseparated grain function $R_{G}$ with respect to the local length as well as the positive derivate of the cumulative sum of the separated grain $S_{G}$

$$\delta_{G} = \frac{dS_{Gi}}{dl} = - \frac{dR_{G}}{dl}$$

For continuous functions, the grade of grain separation is denoted by the separation index $\eta_{iG}$. This index is calculated by relating separated grain quantity for each section to the average available grain quantity in this section i.e. the quotient of local separation rate $\delta_{G}(l_x)$ and local remaining unseparated grain $R_{G}(l_x)$.

$$\eta_{iG} = \frac{\delta_{G}(l_x)}{R_{G}(l_x)}$$

**LITERATURE REVIEW**

Empirical, stochastical and physical approaches are presented to model the cumulated separated grain sum function $S_{G}$, whereas analytical equations are used for kernel motion behav-
iour modelling. Like empirical methods, stochastical and physical approaches need one or more exponential functions to describe the grain separation process.

Single exponential functions

A simple approach for modelling the cumulative sum of the separated grain $S_G$ and remaining unseparated grain $R_G$ is a single exponential function with a constant separation coefficient $\lambda = \text{const.}$:

$$
S_G = 1 - e^{-\lambda t} \\
R_G = e^{-\lambda t} \\
\delta_G = \lambda e^{-\lambda t} \\
\eta_{IG} = \lambda
$$

A constant separation coefficient not always represents the experimental data well enough. Therefore several approaches with a single exponential function but linear exponents are suggested in literature, table 1. Usually the above mentioned separation coefficients are valid for one test conditions only. They depend on crop properties, throughput, design parameters and adjustments of the separation unit.

The influence of peripheral speed and MOG throughput on the threshing coefficient for an axial threshing unit (Figure 4), for example, can be expressed according to Wacker (1985), by a second order polynomial

$$
\lambda = a_1 + b_1 v_R + c_1 v_R^2
$$

Caspers (1973) chose a third order polynomial as exponent for a single exponential function to describe his experimental results on a tangential threshing unit, table 1; Rusanov (1971) proposed an exponent of $a = 0.9$ for the separation length in the exponential function. Maertens and Baerdemaker showed that the equation from Rusanov, table 1, fits best to an experimental data set of New Holland, but this equation is discussed again in Russia, (Zhalnin 2001, Rusanov 2001, 2002, Vetrov 2002). Gregory and Fedler (1987) as well as Kim and Gregory (1991) established single exponential functions for the separation on straw walker and cleaning unit with an exponential function as exponent, table 1.

Single exponential functions are clear, simple, and easy to calculate. But the agreement with experimental results is only satisfying, if the separation rate at the front of the separation unit is very high. This occurs only, if a presegregation on the grain pan of the cleaning unit has already occurred and the grain kernels are already on the bottom of the MOG layer (Figure 5).
A similar effect has a pre-threshing in the feeding device of the threshing unit such that a lot of free, threshed kernels are already available for high separation rate at the front of the concave. For grain in the middle of the MOG layer (Figure 5), or not pre-threshed grain, the separation rate at the front of the separation unit starts with a small value close to zero. This delayed separation, typical for the threshing and separating units, can only be given through extensive functions as exponents. Likewise, there are separation rates found in literature which pass through the origin (delayed separation) or steadily decrease from a high starting value.

**Several exponential functions**

The delayed increase of the cumulative separated grain can also be illustrated through the difference of two exponential functions, table 2. These exponential functions are developed by the separate consideration of threshing or migration and separation (Figure 6).

![Figure 5. Influence of grain distribution on cumulative separated grain $S_G$ and separation rate $\delta_G$.](image)

![Figure 6. Separation model combining grain migration and grain sieve passage](image)
This approach from Böttinger (1993) and similar approaches, for example from Miu (1997,1998) lead to the description of the cumulative separated grain sum function as well as the remaining unseparated grain function by the difference of two exponential functions with the advantage of very good representation of experimental data. Most of the authors chose constant coefficients for threshing or migration and separation (sieve passage) but Böttinger investigated three possibilities for the coefficient $\lambda$. A constant, a linear and a power function with the same exponent D for migration and sieve passage.

$$\lambda_A = \text{const}, \quad \lambda_A = A \cdot 1, \quad \lambda_A = A l^A; \quad \lambda_B = \text{const}, \quad \lambda_B = B \cdot 1, \quad \lambda_B = B l^D$$

These coefficients correspond to the grain separation index $\eta_{\text{G}}$. In particular the power function is very flexible in fitting experimental data, but the values of A and B mostly differ only very little, leading to very big values of each exponential function to get the separation function as difference of both.

Miu (1997,1998) presented a fairly mathematical description of threshing and separation in axial threshing units using one exponential function for threshing and another for separation each with constant threshing coefficient $\lambda$ as well as constant separation coefficient $\beta$. The mathematical model was validated with data from experimental research on two axial threshing units, Wacker (1985). He could describe the influence of several adjustment parameters with the following equations.

$$\beta = k_\beta \cdot p \cdot \sqrt{\frac{v \cdot M_{\text{MOG}} \cdot \sqrt{m_{\text{MOG}}}}{\sqrt{\rho_{\text{MOG}}}}} \cdot e^{-\left(\frac{m_{\text{MOG}} + U_{\text{MOG}}}{m_{\text{MOG}} + U_{\text{MOG}}}ight)}$$

$$\lambda = k_\lambda \cdot \sqrt{\frac{\rho_{\text{MOG}} \cdot v \cdot c_r}{m_{\text{MOG}} \cdot \sqrt{U_{\text{MOG}}}}} \cdot e^{-\left(\frac{\rho_{\text{MOG}} + U_{\text{MOG}}}{\rho_{\text{MOG}} + U_{\text{MOG}}} - \frac{v}{v_o}\right)}$$

$p$: probability of kernel passage  
$v$: peripheral rotor speed [m/s]  
$m_{\text{MOG}}$: MOG throughput [kg/s]  
$U_{\text{MOG}}$: MOG moisture content [%]  
$\rho_{\text{MOG}}$: MOG bulk density  
$c_r$: clearance rear concave [mm]  
$k_\beta$, $k_\lambda$: coefficients  
$m_{\text{MOG}_0}$, $U_{\text{MOG}_m}$, $\rho_{\text{MOG}_o}$, $v_0$: reference values

At the moment the model from Miu is a good compromise between clearness, application and agreement with experimental data. Further development towards the incorporation of design parameters would be helpful.

All exponential functions are a valuable approach for modelling grain separation. Agreement with experimental data can be improved by extended exponential functions, but physical relations are scarcely evident. Influence of crop properties, adjustment and design on the threshing and separation coefficients can be determined from experimental data. Up to now universally applicable coefficients are not available.
Convection Diffusion Model

The convection-diffusion model from Meinel and Schubert (1971), extended by Beck (1999), describes physical processes involved in grain penetrating the MOG layer. The stochastic vertical grain movements are combinations of constant sinking movements (convection) and random scattering (diffusion).

Segregation and separation take place in a volume element of the layer of grain and MOG being transported over the sieve (Figure 7).

![Figure 7. Model of grain/chaff separation on the cleaning shoe considering a volume element being transported over the sieve.](image)

The volume element has small extensions in x- and z-direction but a large extension in y-direction. The y-direction represents the vertical distance from the sieve. In the beginning all grain is assumed to be concentrated on top of the MOG. The volume element is then transported over the sieve with a constant transport velocity $v_x$. During the dwell time on the sieve $T$, which can be calculated from the length of the sieve $l$ and the transport velocity, kernels penetrate the layer during the segregation process. At the end of the sieve most of the grain has been separated.

With the volume element shown in Figure 8 segregation and separation can be described and connected mathematically. Within the layer of MOG segregation is assumed to base on the physical laws of convection and diffusion. The distribution function of the grain mass $u(y,t)$ is determined by the diffusion component with a constant diffusion parameter $D_y$ and by the convection component with an average sinking velocity $v_y$.

![Figure 8. Volume element of the layer of grain and MOG for segregation and separation](image)
Having penetrated the layer of MOG the kernels are separated by the sieve with a time delay. This time delay is mainly determined by the sieve design, the size of the kernels and the width of the sieve openings. Former investigations showed, that the grain separation rate is usually proportional to the grain mass on the surface of the sieve. This leads to a single exponential function, with \( \dot{u}_S(t) \) being the grain separation rate, \( u_A(t) \) being the grain mass on the surface of the sieve and \( T_A \) being the time constant of separation, which reads:

\[
\dot{u}_S(t) = \frac{1}{T_A} u_A(t)
\]

Beck (1999) connected both equations in order to calculate the theoretical grain distribution in the layer, the grain loss and the remaining unseparated grain fraction on the sieve. The model parameters for diffusion \( D_y \), convection \( v_y \) and separation \( T_A \) are approximated by linear equations from the parameters of the cleaning shoe. Since the basic equations are differential equations, initial, as well as boundary conditions have to be specified before they can be solved numerically by applying a Finite Difference Scheme.

One special feature of the simulation model is the possibility to visualize the grain distribution within the layer on the cleaning shoe or grain pan. The basic equation of segregation is solved for the grain distribution function \( u(y,t) \). Results are displayed using visualization tools as shown in Figure 9.

**Figure 9.** Predicted grain distribution in the layer with no chaff being displayed.

Calculations were carried out with model parameters for the standard parameters of the cleaning shoe. In the beginning all grain is concentrated in a thin layer on top of MOG. When segregation starts grain is penetrating the layer towards the sieve. Due to stochastic effects the actual sinking velocity is different for each kernel, which leads to varying penetration times.

Since the grain distribution within the layer is impossible to measure, this part of the simulation model extends the methods of investigations on separation units. This model can be used for straw walkers, grain pans, and cleaning units, whereby the values for sinking speed, dispersion constant, and separation coefficient are different for different separation units and depend on crop properties, adjustment, and design parameters.

For the numerical solution the initial conditions are important i.e. the grain distribution in the MOG layer at the front of the cleaning unit. Thereby this model shows the advantages of pre-segregation and stimulate considerations for improvements of material flow in the combine. To determine the transport velocity \( v_x \) which is needed to calculate the dwell time \( T \) on the sieve, Beck used the analytical model developed by Freye (1980).
Analytical Model

An analytical solution for the grain movement on the cleaning shoe and for theoretical calculations of several factors important for evaluation of the separation process, for example transport velocity, loosening of the layer in the throw phase as well as the gliding distance and gliding speed between the grain and sieve was given by Freye (1980). The two following equations of motion can be derived using the forces shown in Figure 10.

\[ Z_P \cdot F_S = \Delta p \cdot A_S \]

\[ Z_P = \frac{6(1-\varepsilon) \cdot A_S \cdot H}{\pi \cdot d_K^3} \]

\[ \Delta p = c_w \cdot \frac{(1-\varepsilon) \cdot H}{\varepsilon^3} \cdot \frac{\delta_L}{2} \cdot v_{rel}^2 \]

\[ m_K \cdot \frac{d^2 \xi}{dt^2} = -F_R - F_T \cdot \cos \gamma - F_G \sin \alpha + F_S \xi \]

\[ m_K \cdot \frac{d^2 \eta}{dt^2} = -F_N - F_T \cdot \sin \gamma - F_G \cos \alpha + F_S \eta \]

\( \alpha \): sieve inclination  \( \beta \): vibration angle  \( \gamma \): air stream direction  \( m_K \): mass of single kernel  \( F_R \): friction force  \( F_T \): inertial force  \( F_N \): normal force  \( F_S \): pneumatic force

A special problem is the determination of the pneumatic force on the kernel, because the common equation for air resistance for free kernels yields too large values. Therefore Freye divided the total pneumatic force on the bulk layer \( \Delta p \cdot A_S \) by the number of kernels \( Z_p \) in the layer. The layer is thrown up by agitation of the sieve. Segregation occurs during flight, whereas the grain is separated by the later impact on the sieve, see Figure 11.

Though impacts between single kernels are not considered and for the determination of the pneumatic force only a bulk of kernels without MOG is used, the model allows reasonable statements for the evaluation of the separation processes. Applying a discrete element method may further improve this model, Sakaguchi et al. (2001).
Separation

Separation rate, remaining unseparated grain, as well as cumulative separated grain sum can be modelled by the equations proposed by Böttinger (1993), table 2. These equations are easily modified for good agreement with experimental data by the three parameters A, B and D. But these parameters of the model function are not independent and partly compensate each other. Often, the parameters A and B differ only very little, which leads to the fact that the remaining unseparated grain function is being calculated as the difference of two large values, resulting from the 2 exponential functions.

Therefore, Schreiber conducted limit considerations for the equations from Böttinger and found the following relations:

\[
R_G = \frac{A \cdot 1^{(D+1)} + D + 1}{(D + 1)} \cdot e^{-\frac{A}{(D+1)} 1^{(D+1)}}
\]

\[
\delta_G = \frac{A^2 \cdot 1^{(2D+1)}}{(D + 1)} \cdot e^{-\frac{A}{(D+1)} 1^{(D+1)}}
\]

Replacing D by the coefficient C with

\[
C = D + 1
\]

the equations are

\[
R_G = \left(\frac{A}{C} \cdot 1^C + 1\right) \cdot e^{-\frac{A}{C} \cdot 1^C}
\]

\[
\delta_G = \frac{A^2}{C} \cdot 1^{(2C-1)} \cdot e^{-\frac{A}{C} \cdot 1^C}
\]

These equations are of the type \( y = x \cdot e^x \) and model separation processes at the front of the separation unit with good agreement, comparable to Böttingers equations. An advantage is, that only 2 parameters are needed to represent the separation rate as well as the remaining unseparated grain function. The mutual influence between the parameters is thereby reduced.

Experimental data by Zhao (2002) obtained by using the cleaning test stand, see Figure 2, were modelled using Schreibers equations. The measurements were conducted by varying the throughput while keeping all other conditions constant (Figure 12). The remaining unseparated grain function as well as the separation rate were represented by the equations with good agreement. The correlation coefficient is \( r^2 = 0.98 – 0.99 \). Only the steep gradient of the separation rate in the 2nd box at low throughput is not as well predicted.
Figure 12. Unseparated grain $R_G$ and separation rate $\delta_G$ versus separation length $l$ comparison of experimental data and model prediction

The coefficients $A$ and $C$ proceed steadily and can be given in dependency of the throughput by using a second order polynomial (Figure 13). For further review, the equations should be validated with additional experimental data.

![Parameter A and C versus grain throughput $m_G$](image)

**Throughput-Loss-Relationship**

The capacity of combines is measured in tests based on DIN 11390 and ISO 8210 or modified standards. Throughput and losses are measured during harvesting a given distance. Different throughputs are realized at different working velocities. Results are 5 to 7 single data of losses at different throughputs and otherwise same conditions. These single data are summarized to a throughput-loss-relationship. The unavoidable scattering of data results in a deviation of measured and calculated data for the selected function. The combine testing institutes like PAMI, FAT or DLG are using four types of functions in their regression analysis:

1. **Linear function**
   \[ y = ax + b \]
2. **Polynomial**
   \[ y = ax^2 + cx + d \]
3. **Power function**
   \[ y = ax^b \]
4. **Exponential function**
   \[ y = e^{ax^2 + cx} \]

Usually all four functions are compared to the measured data. The function with the best correlation is then used to represent the test results. The throughput at 1% loss and at 2% loss, for
example at DLG-standard, is calculated from the selected function and given as combine capacity.

Nowadays the DLG is using the exponential function only, which can be derived from the single exponential function model for the remaining unseparated grain. For a set of test runs with the threshing test facility at Hohenheim University with fresh wheat variety Ritmo, at different moisture contents and with stored wheat of the same variety the functions agree with experimental data quite well (Figure 14).

While the polynomial function shows a loss increase for low throughputs which does not correspond to the loss characteristic of a tangential threshing system, the exponential function changes its curvature, which is not to be expected for loss characteristics. Besides that, the coefficients do not change continuously with the moisture content.

Schneider et al. (2000) together with Miu proposed an extended exponential function given by:

\[ y = a \cdot x^{-0.5} \cdot e^{bx} \]

Figure 14. Test results of the throughput loss relationship and agreement with several functions which corresponds very well to experimental data (Figure 14). The parameters a and b in this extended exponential function, Figure 15, are the only parameters in the shown functions which are strictly monotone increasing and decreasing, respectively, with the MOG moisture content.
Figure 15. Coefficients a und b for the extended function versus moisture content

This extended exponential function seems to be a better choice for modelling the throughput-loss-relationship compared to the single exponential function. Extended exponential functions should be further validated with different experimental data.

CONCLUSION

Several mathematical models using empirical, statistical, and physical approaches have been developed in the last decades which describe the threshing and separating process of combine harvesters. In simple models, one exponential function is used to represent the cumulative separation function, where the exponent is a constant or dependents on the separation length. The exponent is also depending on crop properties, adjustment, and design parameters.

A better interpretation of the separation delay at the beginning of concave, walker, or sieve is performed by two exponential functions. The coefficients in these functions are also influenced by crop properties, adjustment, and design parameters. However, the whole range of possible influences has to be investigated further. A deep insight into the separation process is possible by the extended convection-diffusion model, which clearly shows the influence of grain distribution in the MOG layer on grain separation. The analytical model describes the motion behaviour of grain kernels on the sieve. Applying a discrete-element method may further improve this model.
<table>
<thead>
<tr>
<th>Reference</th>
<th>Equation</th>
<th>Coeff. for:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vasilenko, 1954</td>
<td>[ R = e^{-b \cdot l} ]</td>
<td>( \lambda ) concave length</td>
</tr>
<tr>
<td>Arnold, 1964</td>
<td>[ S = 1 - e^{-\lambda \cdot l} ]</td>
<td>( \delta ) compression</td>
</tr>
<tr>
<td>Filatov, Chabrat, 1967</td>
<td>[ S = 1 - e^{-(a_1 \cdot l + a_2 \cdot l^2 + a_3 \cdot l^3)} ]</td>
<td>( \psi ) feedrate</td>
</tr>
<tr>
<td>Gubsch, 1969</td>
<td>[ S = 1 - e^{-(\lambda \cdot l + \delta \cdot \psi)} ]</td>
<td>( \beta ) threshing</td>
</tr>
<tr>
<td>Reed, Zoerb, Bigsby, 1974</td>
<td>[ R = e^{-(\lambda \cdot l + \delta \cdot \psi)} ]</td>
<td></td>
</tr>
<tr>
<td>Wacker, 1985</td>
<td>[ R = e^{-\mu \cdot l^{\alpha}} ]</td>
<td></td>
</tr>
<tr>
<td>Caspers, 1973</td>
<td>[ R = e^{-(\varrho \cdot e^{-r \cdot MOG}) \cdot l} ]</td>
<td></td>
</tr>
<tr>
<td>Rusanov, 1971</td>
<td>[ S = 1 - e^{-(a_1 \cdot l + a_2 \cdot l^2 + a_3 \cdot l^3 + a_4 \cdot l^4)} ]</td>
<td></td>
</tr>
<tr>
<td>Pustygin, 1978</td>
<td>[ R = e^{-\psi} ]</td>
<td></td>
</tr>
<tr>
<td>Lo, 1978</td>
<td>[ S = 1 - e^{-k \cdot p_0 \cdot e^{-c_{\psi}}} ]</td>
<td></td>
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<tr>
<td>Trollope, 1982</td>
<td>[ R = e^{\left(\frac{k_3}{d_K} \cdot \frac{B \cdot v_x \cdot \rho_{MOG}}{m_{MOG}} \cdot e^{-k_1 k_2 \cdot \frac{\dot{m}_{MOG}}{B \cdot v_x}}\right) \cdot l} ]</td>
<td></td>
</tr>
<tr>
<td>Gregory, Fedler, 1987</td>
<td>[ R = e^{-\frac{\dot{m}<em>{K}}{m</em>{MOG}} \cdot F \cdot \frac{B \cdot z \cdot n}{E_n m_{ges}} \cdot e^{0.146 + 0.00139 \cdot \frac{1D_s}{9D}} \cdot e^{-0.176 \cdot \frac{1D_s}{9D}}} ]</td>
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<tr>
<td>Gregory, 1988</td>
<td>[ S = e^{-k_1 l (1-e^{-k_2 \cdot l})e^{-k_3}} ]</td>
<td></td>
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<tr>
<td>Kim, Gregory, 1991</td>
<td>[ R = e^{-k_1 l (1-e^{-k_2 \cdot l})e^{-k_3}} ]</td>
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Table 2. Several exponential functions for describing the separation process

<table>
<thead>
<tr>
<th>Authors</th>
<th>Equation</th>
<th>Parameters/Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alferov, Bragince, 1972</td>
<td>( S = 1 - e^{-\lambda_1 t} - X_A \frac{1}{\lambda_1 - \lambda_0} \left( e^{-\lambda_0 t} - e^{-\lambda_1 t} \right) )</td>
<td>( X_A ) fraction of unthreshed grain, ( \lambda_1 ) separation, ( \lambda_0 ) migration, ( \lambda_0 ) separation</td>
</tr>
<tr>
<td>Klenin, Lomakin, 1972</td>
<td>( A = 1 - \frac{X_A}{k_2 \cdot \mu} \left( k_2 e^{\mu t} - e^{k_1 t} \right) \left( 1 - X_A \right) \cdot e^{\mu t} )</td>
<td>( k_2 ), ( \mu ), ( X_A ), ( X_A ) separation</td>
</tr>
<tr>
<td>Chrolikow, 1974</td>
<td>( R = \frac{1}{k_2 - k_1} \left( k_1 e^{-k_2 t} - k_2 e^{-k_1 t} \right) )</td>
<td>( k_1 ), ( k_2 )</td>
</tr>
<tr>
<td>Huyhn, Powell, 1978</td>
<td>( R = \frac{\tau_1 \tau_2}{\tau_2 - \tau_1} \left( \frac{1}{\tau_1} e^{-\tau_1 v_x} - \frac{1}{\tau_2} e^{-\tau_2 v_x} \right) )</td>
<td>( \tau_1 ), ( \tau_2 ), ( v_x )</td>
</tr>
<tr>
<td>Trollope, 1982</td>
<td>( S = 1 + \frac{1}{k_0 p_o - c} \left( c e^{-k_0 p_o \psi} - k_0 p_o e^{-c \psi} \right) )</td>
<td>( k_0 ), ( p_o ), ( \psi )</td>
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<tr>
<td>Huyhn, 1982</td>
<td>( S = 1 - \frac{1}{m} \sum_{j=1}^{m} \frac{1}{(\lambda_1 - \lambda_2)(\lambda_2 - \lambda_3)(\lambda_1 - \lambda_3)} \cdot \left[ \lambda_2 \lambda_3 (\lambda_2 - \lambda_3) e^{-\lambda_1 t} + \lambda_3 \lambda_1 (\lambda_3 - \lambda_1) e^{-\lambda_2 t} + \lambda_1 \lambda_2 (\lambda_1 - \lambda_2) e^{-\lambda_3 t} \right] )</td>
<td>( \lambda_1 ), ( \lambda_2 ), ( \lambda_3 )</td>
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<tr>
<td>Böttinger, 1993</td>
<td>( R = \frac{1}{B - A} \left[ \begin{array}{c} -A e^{D+1} \cdot l^{D+1} \ -B e^{D+1} \cdot l^{D+1} \end{array} \right] )</td>
<td>( B ), ( A ), ( \lambda ), ( l )</td>
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<tr>
<td>Miu, 1995</td>
<td>( S = \frac{1}{\lambda - \beta} \left[ \lambda \left( 1 - e^{-\beta t} \right) - \beta \left( 1 - e^{-\lambda t} \right) \right] )</td>
<td>( \lambda ), ( \beta )</td>
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<tr>
<td>Miu, 1997</td>
<td>( S = \frac{1}{b} \left[ a \left( 1 - e^{-bl} \right) - b \left( 1 - e^{-al} \right) \right] )</td>
<td>( a ), ( b ), ( l ), ( al )</td>
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REFERENCES


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<td>Dieter</td>
<td>Kutzbach</td>
<td>Full professor</td>
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